MATE-1215: Calculo integral – Ecuaciones diferencial

Tarea 4

Problem 1. Let $q \geq 0$. Do the following sequences converges? If they converge, find the limit. Prove your assertions.

(a) $(q^n)_{n\in\mathbb{N}}$, (b) $(\sqrt[n]{n})_{n\in\mathbb{N}}$.

Problem 2. (a) Show that the so-called p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges if $p > 1$ and diverges if $p \leq 1$.

(b) Determine whether the following series are convergent, absolutely convergent or divergent.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \arctan(n)}{n}$, (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{a^{2n}-1}{2^n}$ where $a \in \mathbb{R}$.

Problem 3. Do the following series converge? Prove your assertions.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}}$, (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$,

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$, (d) $\displaystyle\sum_{n=1}^{\infty} \Big(a+\frac{1}{n}\Big)^n$ where $a \in \mathbb{R}$.

Problem 4. (Koch's snowflake curve)

Given a polygon, the middle third of each side of the polygon is removed and an equilateral triangle is attached instead.

If the initial polygon is an equilateral triangle, then the *snowflake curve* is the limit if the procedure described above is iterated inifintely often. Find the circumference and the area of the snowflake if each of the sides of the initial triangle has length $a > 0$. Prove your assertions.